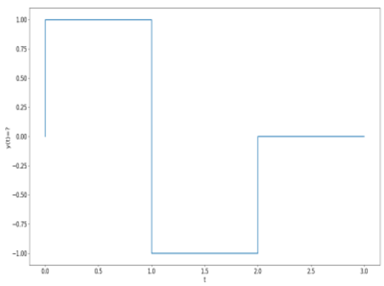
HW Midterm 1: Fall 2018 Name: ID:

Question 1 (20pts.):

Write the equation for the waveform shown below in terms of step and/or ramp functions. Show work.



$$f(t) = u(t) - 2u(t-1) + u(t-2)$$

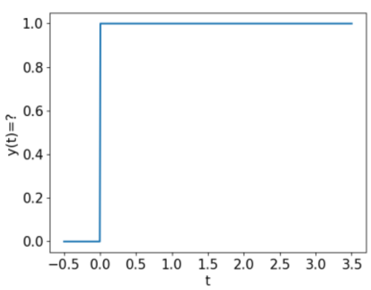


Figure 1: $u(t)$

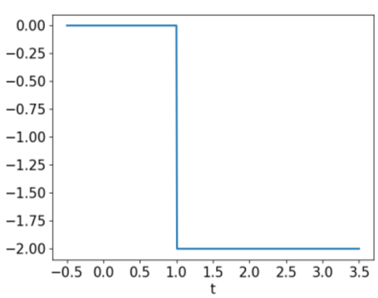


Figure 2: $-2u(t-1)$

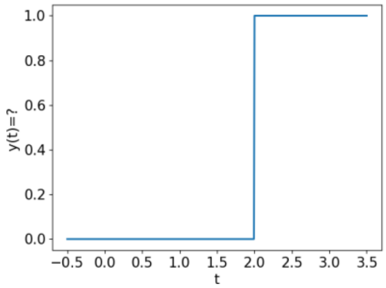


Figure 3: $u(t-2)$

Sum together:

$$f(t) = u(t) - 2u(t-1) + u(t-2)$$

Question 2 (10pts):

Sketch the Bode plots (magnitude (10pts.) and phase (10pts.)) of the following transfer function (Use radians/s.):

$$H(s) = \frac{100s^2}{(s+10)^2}$$

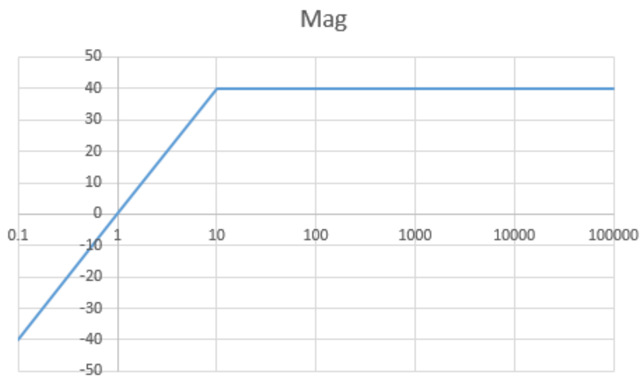


Figure 4: Magnitude of 2'nd order high pass filter (dB) vs. ω (r/s)

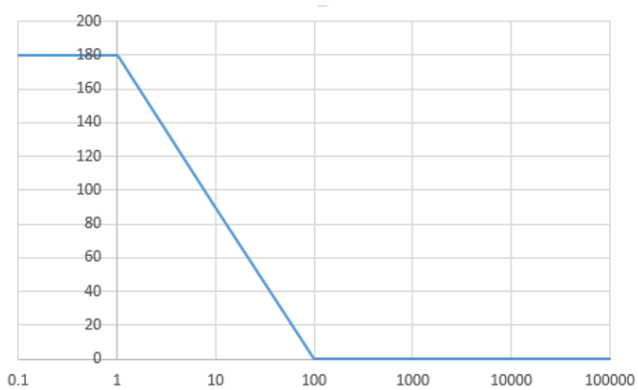


Figure 5: Phase of 2'nd order high pass filter (degrees) vs. ω (r/s)

Question 3 (20pts):

A square wave V(in) is input into two filters with the same RC time constant ($\omega_0 = \frac{1}{RC}$). The transient response of these filters can be seen in figure 1, and the Bode plot of these two filers can be seen in figure 2. As can be seen the node names and waveform colors have been changed so one cannot tell which bode plot belongs to which transient response.

1. Match the transient response to the Bode plot (10pts).
2. Explain your reasoning (10pts).

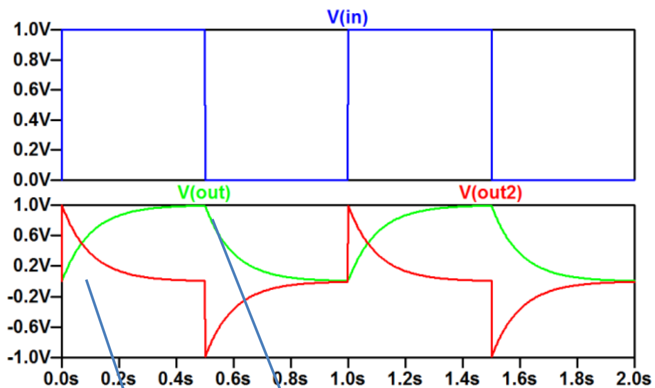


Figure 6: Transient response of two filters. Vs. time.

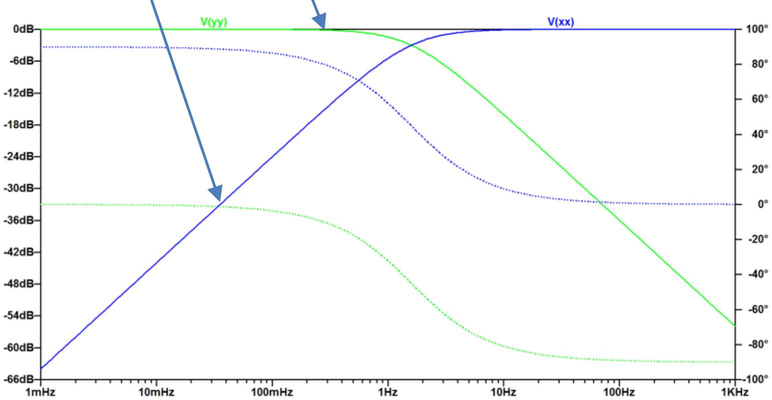


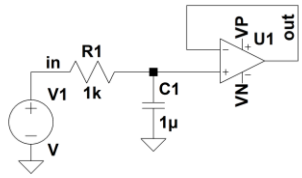
Figure 7: Bode plot of two filters.

The red Bode plot (V(yy)) shows that higher frequencies are attenuated and lower frequencies are passed through with little attenuation. A step function starts with an infinite slope change which is implies a high frequency component of the signal. The low pass filter (V(out) in blue) "blurs" this edge and then passes the 1V DC signal.

The aquamarine Bode plot (V(xx)) shows that lower frequencies are attenuated and higher frequencies are passed through with little attenuation. A step function starts with an infinite slope change which is implies a high frequency component of the signal. The high pass filter (V(out2) in green) detects this edge and then attenuates the 1V DC signal until it reaches zero.

Question 4 (20pts.):

- Derive $H(s)=V_{out}(s)/V_{in}(s)$ for the circuit below (5pts.)
- Sketch the Bode plots (magnitude and phase) of H(s). You can use radians per second. (5pts.)
- Find an equation for the time domain response to a 1V step at t=0s. (5pts.)
- Sketch the time domain step response. Make sure to get the start voltage, end voltage, curve shape, and time scale correct. (5pts.)



Assume ideal OPAMP, with negative feedback and set $V^+ = V^- \text{ and the fact that } V^- = V_{out}$ due to the wire and you can recognize that it is a buffer and that $V_{out} = V^+$

V^+ can be found with voltage division: $\frac{V^+}{V_{in}} = \frac{Z_{C1}}{Z_R+Z_{C1}} = \frac{1}{R_1C_1} \times \frac{1}{s+\frac{1}{R_1C_1}}$ Let $\omega_0 = \frac{1}{R_1C_1} = 1000\left(\frac{r}{s}\right)$

The transfer function becomes: $H(s) = \frac{1000}{s+1000}$

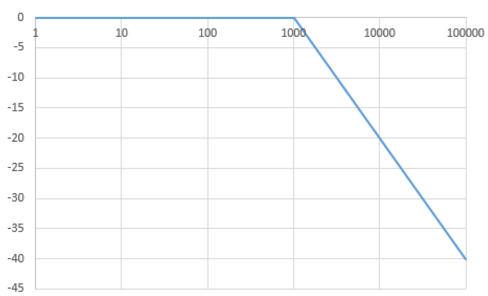


Figure 8: Magnitude of 1'st order low pass filter (dB) vs. ω (r/s)

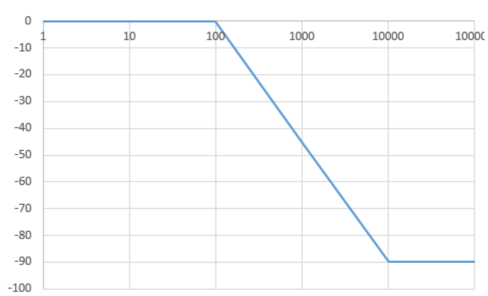
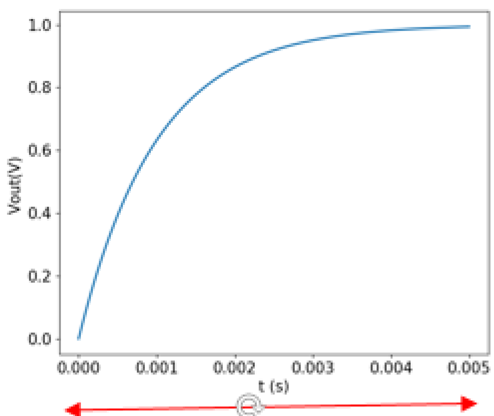


Figure 9: Phase of 1'st order low pass filter (degrees) vs. ω (r/s)

$V_{out}(t) = L^{-1}\left(\frac{1000}{s+1000} \times \frac{1}{s}\right) = 1 - e^{-1000t}$ (From Table)

$$\tau = R_1 C_1 = 1ms$$



$$5\tau = 5ms$$

Question 5 (20pts.):

Find the time domain convolution equation of the two signals shown below and plot that equation showing all relevant parts such as axis labels, max, min values, shape and slopes of the curve:

$$Signal_1 = r(t) - 2r(t-1) + r(t-2)$$

$$Signal_2 = 100\left(u(t) - u\left(t - \frac{1}{100}\right)\right)$$

Signal 1 is non-zero for two seconds, and signal 2 is non-zero for 1/100 of a second. This means that the plot will be non-zero for 2.01 seconds. Looking at signal 2, we see its non-zero area is 1, which when taken in conjunction with signals 2's width being 200 times smaller than signal 1's width, the convolution of the two signals will look like signal 1 convolved with a delta function.

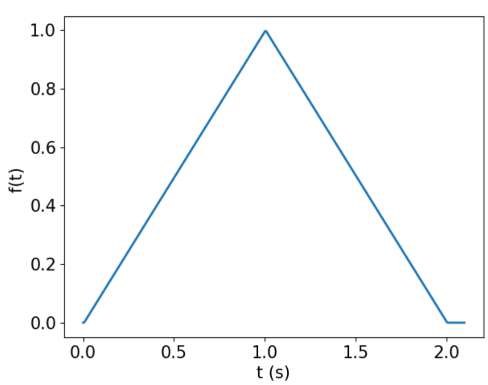
$$f(t) = Signal_1(t) * Signal_1(t) = L^{-1}\big(Signal_1(s) \times Signal_1(s)\big)$$

$$Signal_1(s) = \frac{1}{s^2} - \frac{2}{s^2}e^{-s} + \frac{1}{s^2}e^{-2s}$$

$$Signal_2(s) = 100\left(\frac{1}{s} - \frac{1}{s}e^{-0.01s}\right)$$

$$Signal_1(s) \times Signal_1(s) = 100\left(\frac{1}{s^3} - \frac{1}{s^3}e^{-0.01s} - \frac{2}{s^3}e^{-s} + \frac{2}{s^3}e^{-1.01s} + \frac{1}{s^3}e^{-2s} - \frac{1}{s^3}e^{-2.01s}\right)$$

$$f(t) = 100\left(\frac{r(t)^2}{2} - \frac{r(t-.01)^2}{2} - r(t-1)^2 + r(t-1.01)^2 + \frac{r(t-2)^2}{2} - \frac{r(t-2.01)^2}{2}\right)$$



The reason you cannot see any parabolic shapes are twofold. First the rectangle wave is so thin, that it is active like a $\delta(t)$ function and two if you write out the $r(t-T)^2$ functions in terms of $(t-T)^2u(t-T)$ functions the squared terms can cancel out. For example:

$$r(t)^2 - r(t-1)^2 = t^2u(t) - (t-1)^2u(t-1)$$

For times greater than zero and less than 1, the function is t^2 (everything else is zero due the delayed step function.) At $t \geq 0$, we get $t^2 - t^2 + 2t - 1 = 2t - 1$, whichis linear!